HOW HANDY IS WEIBULL? OR, THE USE OF FAILURE DISTRIBUTIONS IN THE PLANNING AND ANALYSIS OF BATTERY EXPERIMENTS

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Summary

Methods for designing experiments to provide for statistical evaluation are well established. Factorial plans have been used successfully for decades. Methods have more recently become available for incorporating analyses of failure distributions, such as the Weibull. When electrical performance data are identified with modes of failure, additional statistical insights can be achieved. Bode in *Lead–acid Batteries* reviews work on life expectancy testing and notes that "The statistics for lead storage batteries indicate a Weibull distribution ...It is characteristic ...that failure results from aging ..." [1].

The objectives of this extremely brief overview are to show (i) how Weibull results can be obtained completely by hand in a few minutes without lengthy calculations, and (ii) an example of augmenting results and understanding of a previously analyzed statistical experiment by use of Weibull analysis of the failure data.

The benefits of planning experiments to include studies of failure modes and distributions will be more accessible if the engineer or technician understands and does most of the analysis himself, rather than being too heavily dependent upon statisticians or computer programs.

How handy is Weibull?

Our use of failure distribution analysis methods in battery projects began seriously after we were favored at EPRI with a presentation of the methods developed by the Pratt & Whitney Aircraft Group [2]. It is based on common manufacturing problems, and can be tried immediately regardless of the user's level of comprehension (there is never any assurance against misuse regardless of expertise!). To identify the booklet's scope, its Introduction is reproduced as an Appendix. As with other advanced theory and concepts, interpretation and application require substantial plain talk and elementary support by experts. The P&W booklet as a tutorial may not eliminate help from statisticians or considerable self-study effort, but it is easily readable.

The method of plotting utilizes a specially designed graph paper, with corresponding tables of ordinates (Fig. 1 and Table 1). Note that ordinates are based on sample size and median values and include 5% and 95% confidence intervals. The user selects a data set of interest, such as battery cycle

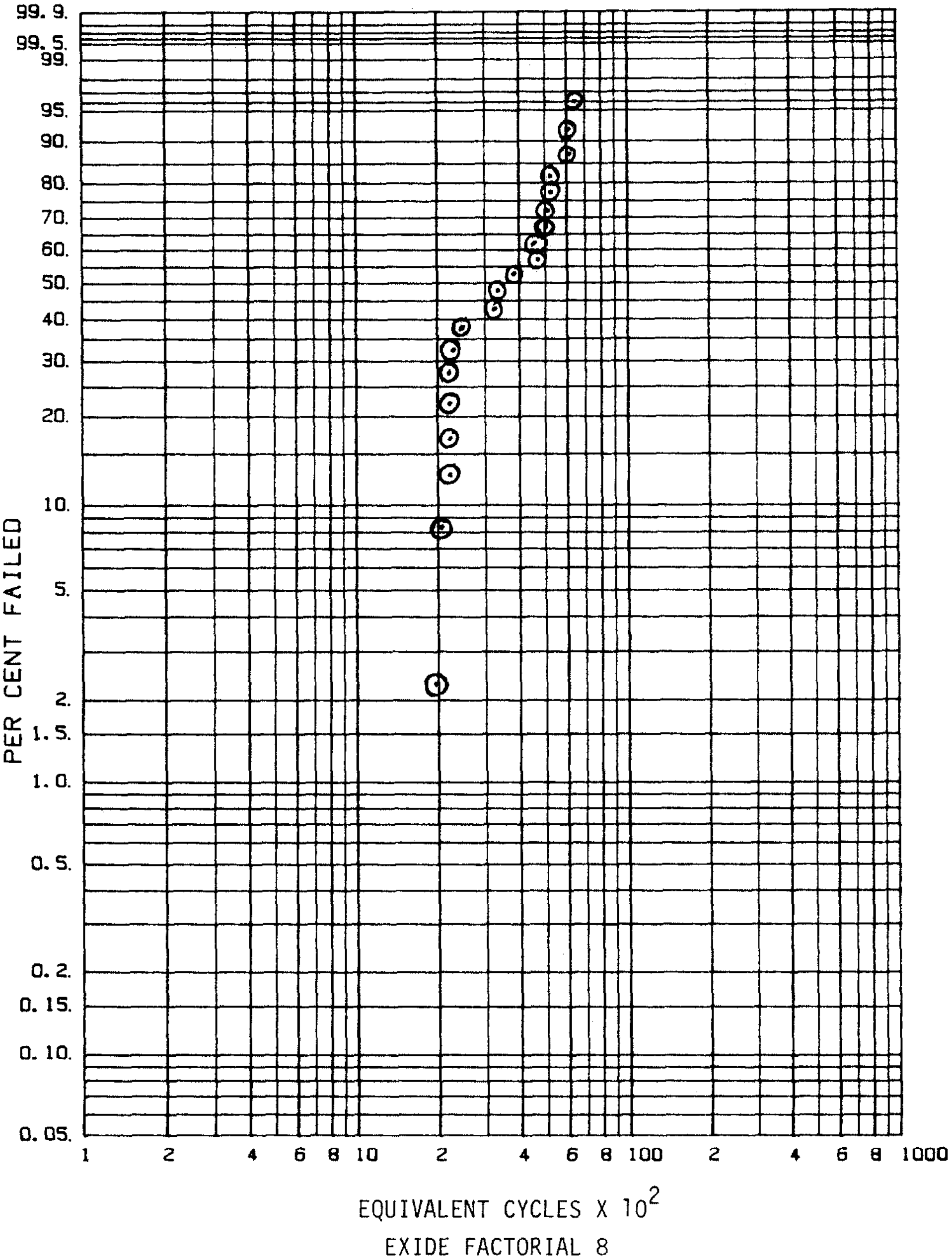


Fig. 1. Handmade Weibull plot.

TABLE 1

Ordinates for P & W Weibull graph paper

Median ranks					Five percent ranks					Ninety-five percent ranks				
Sample size					Sample size					Sample size				
Rank order	8	12	16	20	Rank order	8	12	16	20	Rank order	8	12	16	20
1	8.3	5.6	4.2	3.4	1	0.6	0.4	0.2	0.2	1	31.2	22.0	17.0	13.9
2	20.1	13.5	10.2	8.2	2	4.6	3.0	2.2	1.8	2	47.0	33.8	26.3	21.6
3	32.0	21.6	16.3	13.1	3	11.1	7.1	5.3	4.2	3	59.9	43.8	34.3	28.2
4	44.0	29.7	22.4	18.0	4	19.2	12.2	9.0	7.1	4	71.0	52.7	41.6	34.3
5	55.9	37.8	28.5	22.9	5	28.9	18.1	13.2	10.4	5	80.7	60.9	48.4	40.1
6	67.9	45.9	34.7	27.8	6	40.0	24.5	17.7	13.9	6	88.8	68.4	54.8	45.5
7	79.8	54.0	40.8	32.7	7	52.9	31.5	22.6	17.7	7	95.3	75.4	60.8	50.7
8	91.7	62.1	46.9	37.7	8	68.7	39.0	27.8	21.7	8	99.3	81.8	66.6	55.8
9		70.2	53.0	42.6	9		47.2	33.3	25.8	9		87.7	72.1	60.6
10		78.3	59.1	47.5	10		56.1	39.1	30.1	10		92.8	77.3	65.3
11		86.4	65.2	52.4	11		66.1	45.1	34.6	11		96.9	82.2	69.8
12		94.3	71.4	57.3	12		77.9	51.5	39.3	12		99.5	86.7	74.1
13			77.5	62.2	13			58.3	44.1	13			90.9	78.2
14			83.6	67.2	14			65.6	49.2	14			94.6	82.2
15			89.7	72.1	15			73.6	54.4	15			97.7	86.0
16			95.7	77.0	16			82.9	59.8	16			99.6	89.5
17				81.0	17				65.6	17				92.8
18				86.8	18				71.7	18				95.7
19				91.7	19				78.3	19				98.1
20				96.5	20				86.0	20				99.7

TABLE 2

Exide factorial 8 lead–acid cells

Reference number			Equiv cycles	Design variables				Percent	
Design	Cell	Fail		Wrap	Ret	PAM	Rib	Acid	D.O.D.
1	41	3	2285	—	Vit	3.55	0.8	83	84
2	42	2	2005	PVC	Vit	3.55	0.8	81	84
3	43	8	2500	—	Sly	3.55	0.8	83	84
4	44	4	2300	PVC	Sly	3.55	0.8	81	84
5	45	5	2330	—	Vit	3.97	0.8	82	76
6	46	1	1990	PVC	Vit	3.97	0.8	80	76
7	47	7	2450	—	Sly	3.97	0.8	82	76
8	48	6	2435	PVC	Sly	3.97	0.8	80	76
9	49	16	5200	—	Vit	3.55	2.4	103	84
10	50	20	6700	PVC	Vit	3.55	2.4	101	84
11	51	17	5600	—	Sly	3.55	2.4	103	84
12	52	18	6000	PVC	Sly	3.55	2.4	101	84
13	53	14	4945	—	Vit	3.97	2.4	102	76
14	54	19	6000	PVC	Vit	3.97	2.4	99	76
15	55	13F	4650	—	Sly	3.97	2.4	102	76
16	56	15	5000	PVC	Sly	3.97	2.4	99	76
17	57	9	3415	—	Vit	3.75	1.6	93	80
18	58	11F	3950	PVC	Vit	3.75	1.6	90	80
19	59	10	3530	—	Sly	3.75	1.6	93	80
20	60	12	4820	PVC	Sly	3.75	1.6	90	80
1	2	3	4	5	6	7	8	9	10

F = Failed (operator error), life cycles estimated.

lifetimes, and puts the values in rank order of failure. To illustrate the P&W method here, life cycle data for 20 batteries in a factorial plan have been selected from a lead–acid storage battery project by Exide for ANL [3, 4]. The factorial plan is summarized in Table 2. Referring to the numbers across the bottom of the Table, columns 5 - 8 have the four design variables, columns 9 and 10 are derived values, column 4 has the equivalent cycles based on an acceleration factor representing the effect of testing at an elevated temperature, and columns 1 - 3 contain numbers for the statistical experiment design, the lead–acid cells and the rank order failures. The 20 equivalent cycle lifetimes (col. 4) are plotted in Fig. 1 using the ordinate values in Table 1 for a sample size of 20.

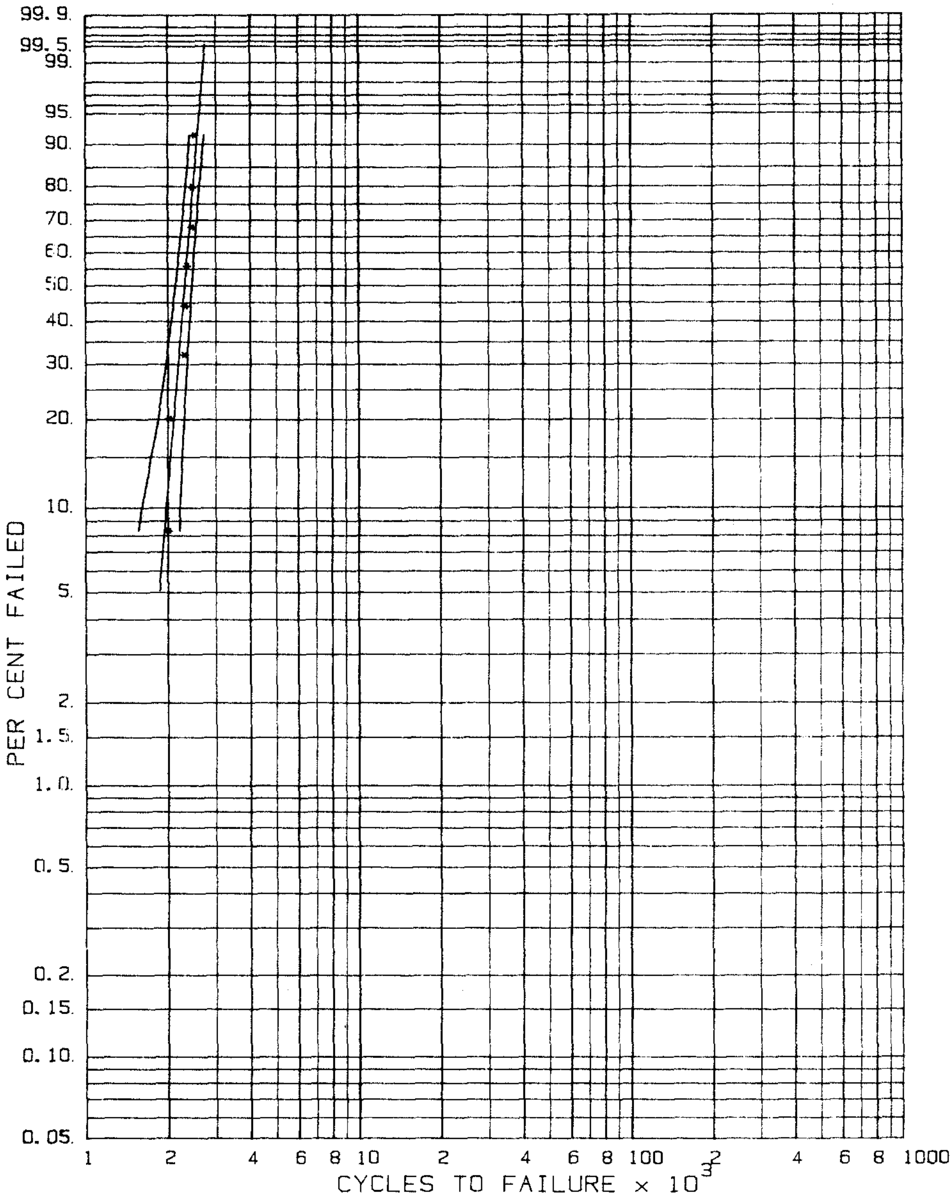
Having made a plot, how to assess it?

The alignment of points in roughly 3 different groups should not be surprising, since the factorial plan shows a logical separation into 3 groups based on the cell rib spacing (col. 8, Table 2). Replotting in subgroups reveals uniform distributions of the failures, 8 values of low rib in Fig. 2 and 8 values of high rib in Fig. 3. Note that these were done by a computer program, having tables of ordinate values stored in the program. It also calculates the associated statistical values of shape factor, mean time to failure, etc. [5]. The hand method is sufficiently accurate, as precise calculations are unnecessary. The shape factor (F) is the slope $\Delta Y/\Delta X$ of the straight line through the points. Characteristic life (A) is the abscissa value at the 63.2% ordinate. Mean time to failure (M) is a fraction of the characteristic life determined by a gamma function: $M = A\Gamma(1 + 1/F)$. The Γ value ranges from 0.886 to 1.

The assumption of rib spacing as the single key variable of significance is also not surprising, as the report by Battelle shows. The analysis of variance in Table 3 for Factorial 8 from the Battelle report also suggests lesser significances for interactions among rib height, wrap, retainer and PAM density. That these possible effects are lost in the plots of Weibull failure distributions is anticipated by the general requirement that real effects in this type of statistically designed experiment will have F test ratios >95%.

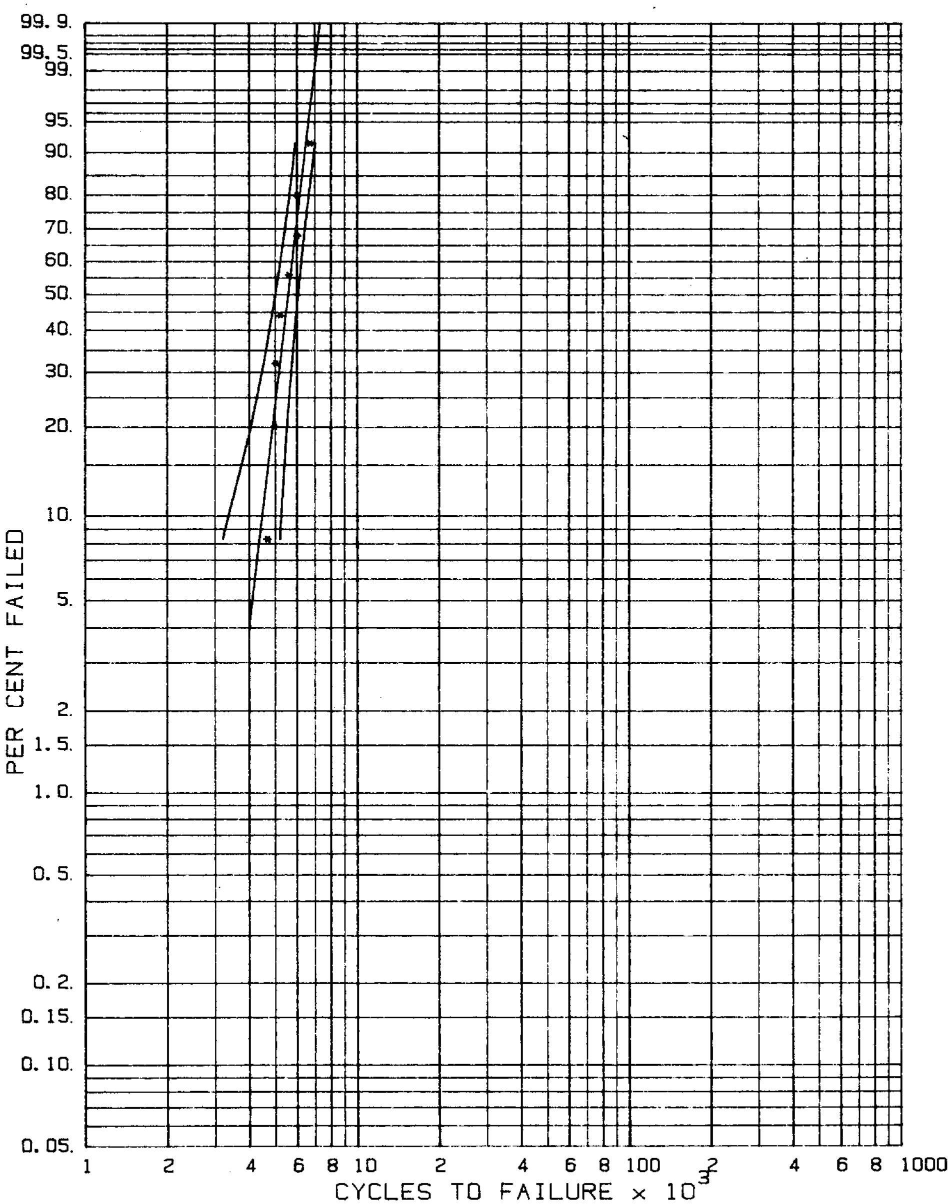
In principle:

Failure distributions should be selected on all significant differences among the properties and performances of the individual items being investigated. In this battery experiment, construction differences are the properties, and cycles to failure are one of the performance factors. If modes of failure had been significantly different, another subdivision of the data set would have been necessary, but all cells failed due to loss of capacity from positive grid corrosion and general degradation of plates. Among the con-



WEIBULL SHAPE PARAMETER: 11. 925
MEAN TIME TO FAILURE: 2280. 4 CYCLES
STANDARD DEVIATION: 232. 20 CYCLES
CHARACTERISTIC LIFE: 2380. 2 CYCLES
CORRELATION COEFFICIENT: 0. 95231
SAMPLE SIZE: 8
NUMBER OF FAILURES: 8

Fig. 2. Exide factorial 8 (low rib) 8 of 8 lifetimes.



WEIBULL SHAPE PARAMETER: 8.4976
MEAN TIME TO FAILURE: 5494.8 CYCLES
STANDARD DEVIATION: 770.50 CYCLES
CHARACTERISTIC LIFE: 5818.0 CYCLES
CORRELATION COEFFICIENT: 0.95461
SAMPLE SIZE: 8
NUMBER OF FAILURES: 8

Fig. 3. Exide factorial 8 (high rib) 8 of 8 lifetimes.

TABLE 3

One-way analysis of variance for factorial 8

Cycle life *vs.* variables	Observed F-ratio	Degrees of freedom (υ_1, υ_2)	Probability of statistical significance
Rib height	73.67	(2, 17)	1.00
Wrap	0.37	(1, 18)	0.45
Retainer	0.00	(1, 18)	0.00
PAM density	0.09	(2, 17)	0.09
Rib height = 0.8 mm			
Wrap	2.96	(1, 6)	0.86
Retainer	7.27	(1, 6)	0.96
PAM density	0.04	(1, 6)	0.15
Rib height = 2.4 mm			
Wrap	4.19	(1, 6)	0.91
Retainer	0.84	(1, 6)	0.61
PAM density	2.79	(1, 6)	0.85

struction differences in the SDE, only the acid ratio (rib height) was seen to be a highly significant variable. The Weibull replots, based on the 2 groups of high and low acid ratio, show two consistent, uniform failure distributions, thus confirming the significance of this variable. Note that failure distributions based on accelerated test data may be skewed toward a greater wearout characteristic than would be found in real use [6].

In conclusion:

What may be inferred from these results and concepts when planning new experiments or purchases of batteries? First, the "hand-y" method of Weibull analysis, and the powerful features available by using failure distribution analysis throughout the course of an experiment (features not included in this brief review) can be an independent asset to any statistically designed experiment or manufacturing quality control program. If minor effects, often assumed to be significant from statistical studies, are actually random effects, then the simplification possible in design, development, and manufacturing could result in considerable savings in time and cost. Finally, a plea for more technical interaction at managerial levels. Statistical methods need to be made more understandable and available outside the province of professional statisticians. If the planning, evaluation, and reporting includes management participation, communication and application of useful results can be more rapidly and meaningfully accomplished.

References

1 H. Bode, *Lead–Acid Batteries*, translation by Brodd and Kordesch, Wiley, New York, 1977.
2 *Weibull Analysis Handbook*, Pratt and Whitney Aircraft, *AFWAL-TR-83-2079, 11-83.*
3 Research, development and demonstration of advanced lead–acid batteries for utility load leveling, *Exide Final Rep., ANL/OEPM-83-6, 8-83.*
4 Analysis of lead–acid battery deep-cycle accelerated testing data, *Battelle Columbus Laboratories Final Rep., SAND84-7105, 6-84.*
5 J. Fatula, Weibull failure distribution analysis and plotting system, *EPRI EM-3658-CCM, 10-84.*
6 C. Luri and A. Steen, Reliability modeling of high voltage batteries, *TRW Space and Technology Group, 17th IECEC Paper #829129, 8-82.*

Additional recommended reading

7 R. Billinton and R. Allan, *Reliability Evaluation of Engineering Systems: Concepts and Techniques*, Plenum Press, New York, 1983.
8 W. Spindler, Cycling up the battery life path with a power boost from statistical distributions, *EPRI, EVC EXPO '83, Paper #8335, 10-83.*
9 S. Basin and W. Spindler, Using statistically designed experiments in development of advanced battery systems, *EPRI EM-1346, 2-80.*

Appendix

WEIBULL ANALYSIS HANDBOOK

AFWAL-TR-83-2079

November 1983

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The objective of this handbook is to provide an understanding of both the standard and advanced Weibull techniques that have been developed for failure analysis. The authors intend that their presentation be such that a novice engineer can perform Weibull analysis after studying this document.

As treated herein, Weibull analysis application to failure analysis includes:

- Plotting the data
- Interpreting the plot
- Predicting future failures
- Evaluating various plans for corrective actions
- Substantiating engineering changes that correct failure modes.

Data problems and deficiencies are discussed with recommendations to overcome deficiencies such as:

- Censored data
- Mixtures of failure modes
- Nonzero time origin ($t_0$correction)
- No failures
- Extremely small samples
- Strengths and weaknesses of the method.

Statistical and mathematical derivations are presented in Appendices to supplement the main body of the handbook. There are brief discussions of alternative distributions such as the log normal. Actual case studies of aircraft engine problems are used for illustration. Where problems are presented for the reader to solve, answers are supplied. The use of Weibull distributions in mathematical models and simulations is also described.